FLUID MECHANICS
(BTME 301-18)

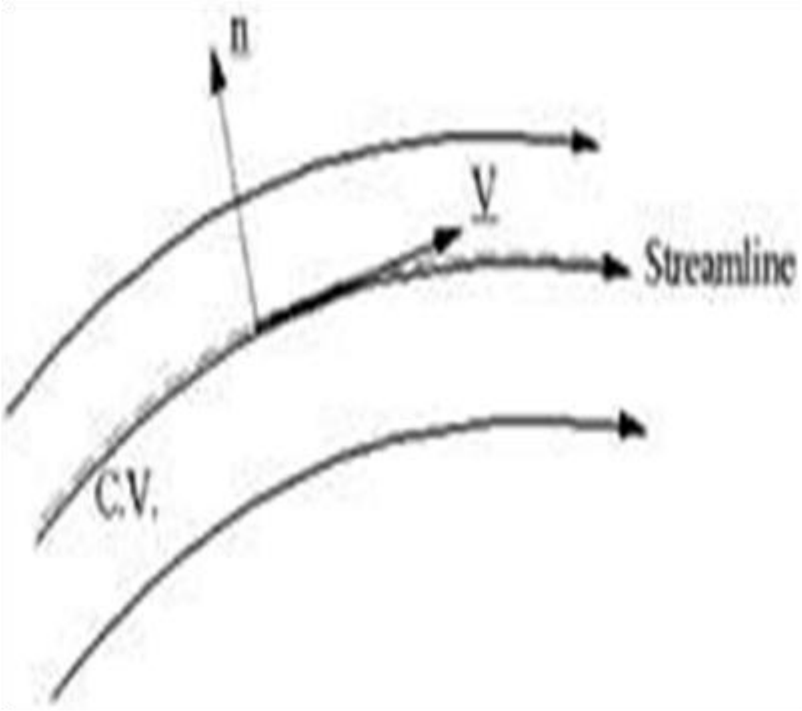
Unit 3: Fluid Kinematics

FLUID KINEMATICS

- Kinematics is defined as a branch of science which deals with motion of particles without considering the forces causing the motion.
- The velocity at any point in a flow field at any time is studied in this.
- Once the velocity is known, then the pressure distribution and hence the forces acting on the fluid can be determined.

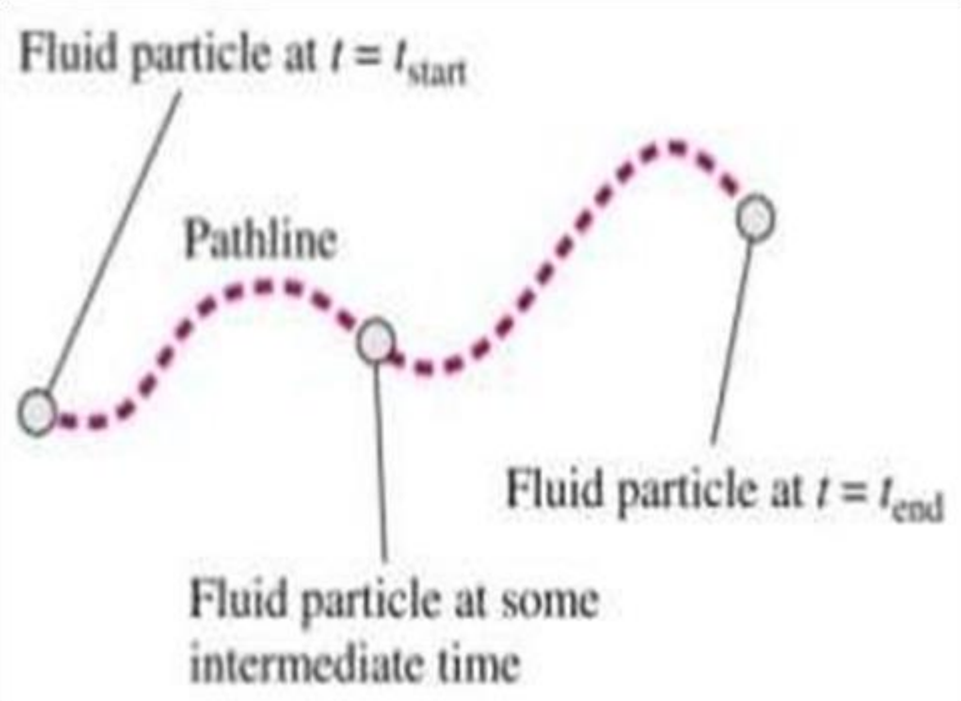
STREAM LINE

- A stream line is an imaginary line drawn in a flow field such that the tangent drawn at any point on this line represents the direction of velocity vector.
- From the definition it is clear that there can be no flow across stream line.
- Considering a particle moving along a stream line for a very short distance 'ds' having its components dx , dy and dz, along three mutually perpendicular co-ordinate axes.
- Let the components of velocity vector Vs along x, y and z directions be u, v and w respectively.



PATH LINE

- A path line is locus of a fluid particle as it moves along.
- In other words a path line is a curve traced by a single fluid particle during its motion.
- A stream line at time t_1 indicating the velocity vectors for particles A and B.
- At times t_2 and t_3 the particle A occupies the successive positions.
- The line containing these various positions of A represents its **Path line**



STREAK LINE

- When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path fallowed by dye or smoke is called the **streak line**.
- Thus the streak line connects all particles passing through a given point.
- In steady flow, the stream line remains fixed with respect to co-ordinate axes.
- Stream lines in steady flow also represent the path lines and streak lines.
- In unsteady flow, a fluid particle will not, in general, remain on the same stream line (except for unsteady uniform flow).
- Hence the stream lines and path lines do not coincide in unsteady non-uniform flow.

STREAK LINE

- When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path fallowed by dye or smoke is called the **streak line**.
- Thus the streak line connects all particles passing through a given point.
- In steady flow, the stream line remains fixed with respect to co-ordinate axes.
- Stream lines in steady flow also represent the path lines and streak lines.
- In unsteady flow, a fluid particle will not, in general, remain on the same stream line (except for unsteady uniform flow).
- Hence the stream lines and path lines do not coincide in unsteady non-uniform flow.

CLASSIFICATION OF FLOWS

- The fluid flow is classified as:
  - Steady and unsteady flows.
  - Uniform and Non-uniform flows.
  - Laminar and Turbulent flows.
  - Compressible and incompressible flows.
  - Rotational and Irrotational flows.
  - One, two and three dimensional flows.

STEADY & UNSTEADY FLOW

- Steady flow is defined as the flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.

- Un-Steady flow is the flow in which the velocity, pressure, density at a point changes with respect to time

UNIFORM & NON-UNIFORM FLOW

- Uniform flow is defined as the flow in which the velocity at any given time does not change with respect to space. (i.e. the length of direction of flow)

- Non-uniform is the flow in which the velocity at any given time changes with respect to space.

LAMINAR & TURBULENT FLOW

- Laminar flow is defined as the flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight and parallel.
- Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called streamline flow or viscous flow.
- Turbulent flow is the flow in which the fluid particles move in a zigzag way.
- Due to the movement of fluid particles in a zigzag way, the eddies formation takes place, which are responsible for high energy loss.

COMPRESSIBLE & INCOMPRESSIBLE FLOW

- Compressible flow is the flow in which the density of fluid changes from point to point or in other words the density is not constant for the fluid.
- For compressible flow, $\rho \neq$ Constant.
- In compressible flow is the flow in which the density is constant for the fluid flow.

ROTATIONAL & IRROTATIONAL FLOW

- Rotational flow is a type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

- And if the fluid particles, while flowing along stream lines, do not rotate about their own axis, the flow is called Irrotational flow.

ONE DIMENSION FLOW

- **1D flow** is a type of flow in which flow parameter such as velocity is a function of time and one space co-ordinate only, say ‘*x*’.
- For a steady one- dimensional flow, the velocity is a function of one space co-ordinate only.
- The variation of velocities in other two mutually perpendicular directions is assumed negligible.
- Hence for one dimensional flow **u = f(x), v = 0 and w = 0**

Where u, v and w are velocity components in x, y and z directions respectively.

TWO DIMENSION FLOW

- **2D flow** is the type of flow in which the velocity is a function of time and two space co-ordinates, say x and y.
- For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.
- The variation of velocity in the third direction is negligible.
- Thus for two dimensional flow **$u = f_1$ (x, y), v = f_2 (x, y) and w = 0.**
- **3D flow** is the type of flow in which the velocity is a function of time and three mutually perpendicular directions.
- But for a steady three-dimensional flow, the fluid parameters are functions of three space co-ordinates (x, y, and z) only.
- Thus for **three- dimensional flow $u = f_1$ (x, y, z), v = f_2 (x, y, z), z = f_3 (x, y, z).**

RATE OF FLOW OR DISCHARGE (Q)

- It is defined as the quantity of a fluid flowing per second through a section of pipe or channel.
- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of the liquid flowing cross the section per second. or compressible fluids.
- The rate of flow is usually expressed as the weight of fluid flowing across the section.
- Thus (i) For liquids the unit of Q is m^3/sec or Litres/sec.

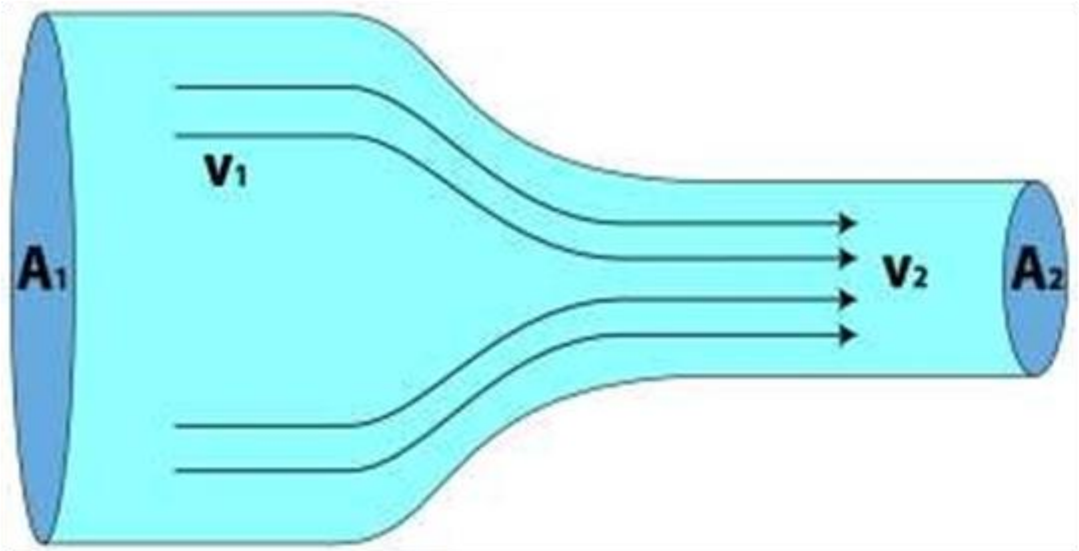
  (ii) For gases the unit of Q is Kgf/sec or Newton/sec.
- **The discharge Q = A×V**

  Where, A = Area of cross-section of pipe.

  V= Average velocity of fluid across the section.

EQUATION OF CONTINUITY

- The equation based on the principle of conservation of mass is called Continuity equation.
- Thus for a fluid flowing through the



- Then the rate flow at section 1-1= $\rho_1 A_1 V_1$
- Rate of flow at section 2-2 = $\rho_2 A_2 V_2$
- According to law of conservation of mass, Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

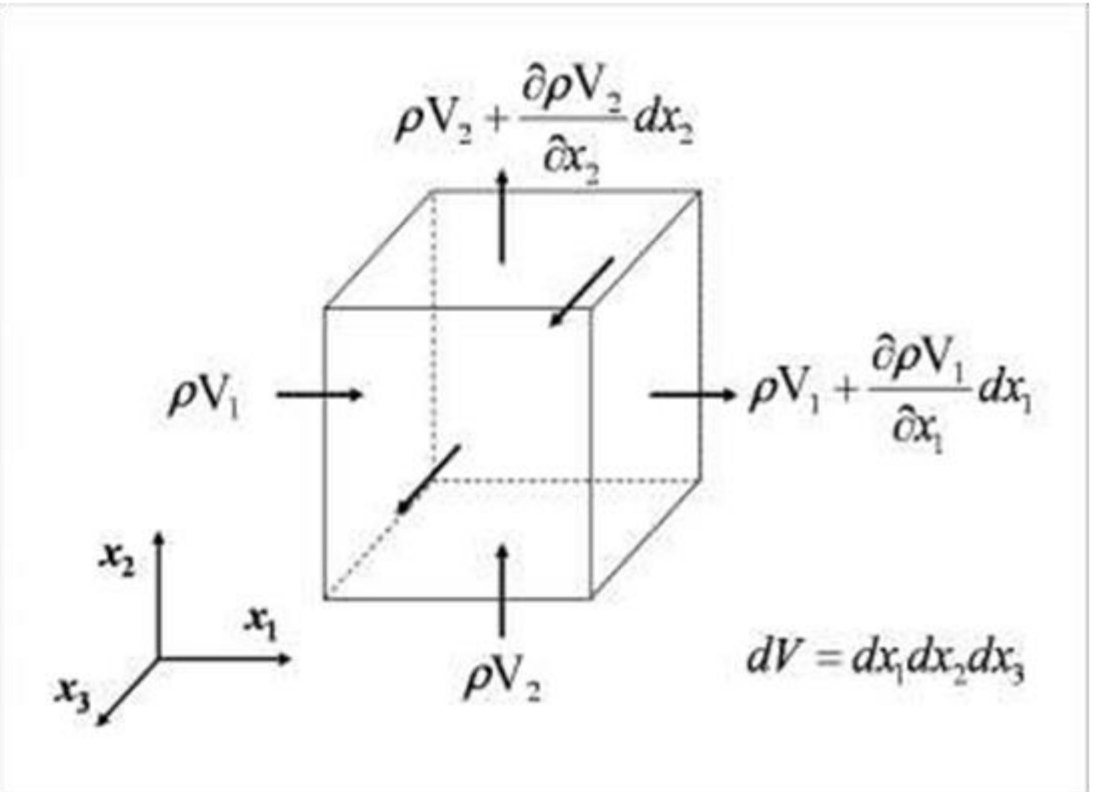
- This equation is applicable to the compressible as well as incompressible fluids and is called **"Continuity equation"**.
- If the fluid is incompressible, then $\rho_1 = \rho_2$ and the continuity equation reduces to

$$A_1 V_1 = A_2 V_2$$

EQUATION OF CONTINUITY FOR 3D FLOW

- Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z.
- Let u, v and w are the inlet velocity components in x, y and z directions respectively.
- Mass of fluid entering the face ABCD per second = ρ × velocity in x - direction × Area of ABCD

  $= \rho \times u \times (dy \times dz)$



- Then the mass of fluid leaving the face EFGH per second

$$= \rho \times u \times (dy \times dz) + \frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$$

- Gain of mass in x- direction = Mass through ABCD - Mass through EFGH per second = $\rho\, u\, dy\, dz - \rho\, u\, dy\, dz - \frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$

$$= -\frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$$

$$= -\frac{\partial}{\partial x}(\rho\, u)dxdydz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

- Similarly gain of mass in y- direction

$$= -\frac{\partial}{\partial y}(\rho v)\ dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

- Similarly gain of mass in z- direction

$$= -\frac{\partial}{\partial z}(\rho w)dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

- Then the mass of fluid leaving the face EFGH per second

$$= \rho \times u \times (dy \times dz) + \frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$$

- Gain of mass in x- direction = Mass through ABCD - Mass through EFGH per second = ρ u dy dz - ρ u dy dz - $\frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$

$$= -\frac{\partial}{\partial x}(\rho\, u\, dy\, dz)dx$$

$$= -\frac{\partial}{\partial x}(\rho\, u)dxdydz \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

- Similarly gain of mass in y- direction

$$= -\frac{\partial}{\partial y}(\rho v)\ dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

- Similarly gain of mass in z- direction

$$= -\frac{\partial}{\partial z}(\rho w)dx\ dy\ dz \quad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

- Net gain of mass = $-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]$ dxdydz__ (4)
- Since mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.
- But the mass of fluid in the element is ρ dxdydz and its rate of increase with time is $\frac{\partial}{\partial t}(\rho.\ dx.\ dy.\ dz.)$ or $\frac{\partial \rho}{\partial t}.\ dx.\ dy.\ dz.$ __(5)
- Equating the two expressions (4) & (5)

$$-\left(\frac{\partial}{\partial}(\rho u) + \frac{\partial}{\partial}(\rho v) + \frac{\partial}{\partial}(\rho w)\right)dx\,dydz = \frac{\partial \rho}{\partial}.dx.dy.dz.$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad \text{\_\_\_\_\_ (6)}$$

- This equation is applicable to
  - Steady and unsteady flow
  - Uniform and non- uniform flow , and
  - Compressible and incompressible flow.
- For steady flow $\frac{\partial \rho}{\partial t} = 0$ and hence equation (6) becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad \text{———(7)}$$

- If the fluid is incompressible, then ρ is constant and the above equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{———(8)}$$

- This is the continuity equation in three - dimensional flow.